Reply to "Comment on 'Dynamic properties in a family of competitive growing models' "

Claudio M. Horowitz and Ezequiel V. Albano
Instituto de Investigaciones Fisicoquímicas Teóricas y Aplicadas (INIFTA), CONICET, CCT-La Plata CONICET, Universidad Nacional de La Plata, Sucursal 4, CC 16, 1900 La Plata, Argentina
(Received 18 September 2009; published 9 March 2010)

In an early paper [C. M. Horowitz and E. V. Albano, Phys. Rev. E **73**, 031111 (2006)], we studied growing models, generically called X/RD, such that a particle is attached to the aggregate with probability p following the mechanisms of a generic model X and at random (random deposition) with probability $(1-p)$. We also formulated scaling relationships that are expected to hold in the limits $p\rightarrow 0$ and $L\rightarrow\infty$, where L is the sample side. In the previous comment, Kolakowska and Novotny (KN) state that our scaling hypothesis does not hold. Here, we show that the criticisms of KN are outlined by analyzing data out of the proper scaling regime and consequently they are groundless and can be disregarded.

DOI: 10.1103/PhysRevE.81.033102 PACS number(s): 05.40.−a, 68.35.Ct

In the previous paper, Kolakowska and Novotny (KN) [1] comment on our early paper (see Ref. [2]). Our paper, as well as a series of previous papers on a related issue [3–5], addresses the properties of a wide variety of growing models, generically called X/RD, involving the deposition of particles according to competitive processes, such that a particle is attached to the aggregate with probability p following the mechanisms of a generic model X that provides the correlations and at random (random deposition, RD) with probability $(1-p)$. A related study on that topic has also been published by Braunstein and Lam [6].

The comments of KN are based essentially on two statements, both of them erroneously attributed to us, which read as follows:

(S1a) "The claim is made that at saturation, the surface width $w(p)$ obeys a power-law scaling $w(p)\propto 1/p^{\delta}$, where δ is only either $\delta=1$ or $\delta=1/2$, which is illustrated by the models where X is ballistic deposition and where X is RD with surface relaxation" (taken from the abstract of the comment of KN [1]). A slightly different version of this statement can also be found in the comment of KN [1], few lines below Eq. (1), namely:

(S1b) "The new claim that is being made in Ref. [1] (i.e. Ref. [2]) is that a nonuniversal and *model-dependent* exponent δ in Eq. (1) must be only of two values, either $\delta=1$ or $\delta=1/2$, for models studied in Ref. [1] (i.e. Ref. [2])."

On the other hand, the second statement, taken from the abstract of the comment of KN [1], reads:

(S2) "Another claim is that in the limit $p\rightarrow 0$, for any lattice size L, the time evolution of $w(t)$ generally obeys the scaling $w(p)\propto(L^{\alpha}/p^{\delta})F(p^{2\delta}t/L^{z})$, where F is Family-Vicsek universal scaling function."

Concerning both (S1a) and (S1b), our answer is that the statement is taken out of context from our paper, so it is incomplete and leads the reader to confusion. In fact, in our paper, we explicitly state in many places that the values $\delta=1$ or $\delta=1/2$ only "hold" in the $p\rightarrow 0$ limit. In fact, in our paper, we state in the abstract, the paragraph before the title of Sec. V, in the title of Sec. V, and in the conclusions that the universality in the δ and y exponents should "hold" in the $p\rightarrow 0$ limit. Also, we have theoretically found those values of δ in the "$p\rightarrow 0$" limit by using a correspondence between two neighboring sites in the discrete model $\{[h(i)-h(i+1)]\}$ and two types of random walks.

Within this context, it is worth mentioning that Braunstein and Lam [6] determined exactly that $\delta=1/2$ for a competitive model between ballistic deposition (BD) and RD, while they found $\delta=1$ for the competition between random deposition with surface relaxation (RDSR) and RD. Of course, Braunstein and Lam [6] clearly stated that the "scaling exponents derived are exact for $p\rightarrow 0$." They also recall that "at finite p, we expect deviation from the exact scalings which indeed were observed numerically in Ref. [4]." See the last paragraph of Sec. II, page 2.

Figures 1(a) and 1(b) of the comment of KN [1] show the behavior of various models, with $X\equiv RDSR$, $X\equiv$BD, and some variants. Results are shown for a wide range of p, actually most results correspond to $p\geq 0.3$, i.e., far away from the correct scaling regime given by the $p\rightarrow 0$ limit. Subsequently, by describing the figures, they state that "in special cases, an approximate power law $w(p)\propto 1/p^{\delta}$ may be observed, however, this is not a principle." From our point of view, it is obvious that one would not expect a nice power-law fit of the data within the whole range of p, but "only in the limit of $p\rightarrow 0$." Also, a careful inspection of Figs. 1(a) and 1(b) of the comment nicely shows that the power law predicted in our paper fits very well the data for $p\rightarrow 0$. So, the data shown in Fig. 1 of the comment of KN [1] seem to be correct, but they are far away from the right scaling limit. The figure clearly suggests that for data taken correctly, namely, for $p\ll 0.1$ as stated in our paper [2], they would certainly obey the proposed scaling behavior.

In a related context, also at the end of the section "Saturation," KN stated that [1] "The other two examples shown in Fig. 1 defy a linear fit. In these cases there is no power law of the type claimed in Ref. [1]" (i.e., Ref. [2]). This absence of power-law scaling in p is also evident in Fig. 4 of Ref. [1] (i.e., Ref. [2]). We notice that in contrast to that opinion, an excellent power-law behavior can be observed in Fig. 4 of our paper [2], but of course in the right scaling regime, namely, for $p\rightarrow 0$.

Concerning (S2), our answer is also that the statement is incomplete and incorrectly formulated, so it leads the reader to confusion. In fact, in Ref. [2], we clearly stated that S2 only holds, as is well known for the standard scaling relations, in the $L \to \infty$ limit, but not "for any lattice size L," as in the misleading statement of KN. The right scaling regime for the sample size is stated in many places of our paper, e.g., along the explanations of Sec. II, as well as the explicit scaling indications written in Eqs. (6)–(8) [2]. We only mentioned in Sec. III that there is numerical evidence of negligible finite-size corrections to the value of the exponent δ. Obviously, this property of δ is not equivalent to the statement formulated by KN in a general way.

All discussed issues and the remaining topics of the comment lead us to disagree with KNs understanding of the concept of scaling in the $p \to 0$ and $L \to \infty$ limits. For example, in the section "The RD limit," another comment of KN reads: "Another claim of Ref. [1] is that Eq. (1) with the power-law prefactors p^{δ} where (δ=1 or 1/2) would prevail in the limit $p \to 0$ and that such a scaling would be universal. We tested these claims in simulations of RD+BD models and found the evidence to the contrary (Figs. 2–3)." Again, Figs. 2 and 3 of the comment of KN show the behavior of RD+BD models using p in the range [0.1, 1], which of course is not at all close to the limit $p \to 0$.

We notice that it is easy to understand why KN claim that our scaling did not fit the data very well. This is the obvious result that one may always obtain just by working with data taken out of the scaling regime but using values for the exponents corresponding to the correct regime. In fact, as we have already shown in our paper, as well as in previous work [3,4], when using small samples, data collapse can only be obtained by using the effective values of the exponents according to the range of p and L used.

In summary, our paper shows that for some models and within the $p \to 0$ and $L \to \infty$ limits, the proposed scaling is a universal principle. On the other hand, in their comment, KN showed that data taken out of the correct scaling limit may depart from scaling.

We acknowledge financial support from the Argentinian Science Agencies CONICET and ANPCyT, and the UNLP.

[1] A. Kolakowska and M. A. Novotny, preceding paper, Phys. Rev. E **81**, 033101(2010).
[2] C. M. Horowitz and E. V. Albano, Phys. Rev. E **73**, 031111 (2006).
[3] C. M. Horowitz, R. A. Monetti, and E. V. Albano, Phys. Rev. E **63**, 066132 (2001).
[4] C. Horowitz and E. V. Albano, J. Phys. A **34**, 357 (2001).
[5] C. Horowitz and E. V. Albano, Eur. Phys. J. B **31**, 563 (2003).
[6] L. A. Braunstein and C.-H. Lam, Phys. Rev. E **72**, 026128 (2005).